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FOUR YEAR B.Sc. (Honours) DEGREE EXAMINATION, NOVEMBER/DECEMBER 2025.

FIFTH SEMESTER

Mathematics

Paper 13 — VECTOR CALCULUS

(w.e.f. 2023 – 24 Regulations)

Time : Three hours

Maximum : 70 marks

(No additional sheet will be supplied)

SECTION A — (5 × 4 = 20 marks)

Answer any FIVE questions.

1. Evaluate $\int_0^2 \int_0^3 xy \, dx \, dy$.

2. Evaluate $\int_0^2 \int_1^x x \, y^2 \, dx \, dy$.

3. Evaluate $\int_0^2 \int_1^3 \int_1^2 x \, y^2 \, z \, dx \, dy \, dz$.

4. Evaluate $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx \, dy \, dz$.

5. If $\phi = 2x^3 y^2 z^4$ find $div(grad\phi)$.

6. Find the maximum value of the directional derivative of the function $\phi = 2x^2 - y - z^4$ at $(2, -1, 1)$.

7. If $F(t) = t\bar{i} + (t^2 - 2t)\bar{j} + (3t^2 + 3t^3)\bar{k}$ then find $\int_0^1 F(t) dt$.

8. If $F = 3xy\,\bar{i} - 5z\,\bar{j} + 10x\,\bar{k}$ evaluate $\int_C F \cdot dr$ along the curve $x = t^2, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

9. If $F = ax\,\bar{i} + by\,\bar{j} + cz\,\bar{k}$ and a, b, c are constants show that $\int F \cdot N dS = \frac{4}{3}\pi(a + b + c)$ where S is the surface of the unit sphere.

10. Apply Gauss's theorem to prove that $\int_S r \cdot N \, dS = 3V$.

SECTION B — ($5 \times 10 = 50$ marks)

Answer ALL the following questions.

11. Evaluate $\iint xy(x+y)\,dx\,dy$ over the region R bounded by $y = x^2$ and $y = x$.

Or

12. Evaluate $\int_0^{\frac{\pi}{4}}\int_0^{a} \sin\theta \frac{r}{\sqrt{a^2 - r^2}}\,dr\,d\theta$.

13. Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y} dx\,dy\,dz$.

Or

14. Using spherical polar coordinates find the volume of sphere $x^2 + y^2 + z^2 = a^2$.

15. If $f = x^2 y\,\bar{i} - 2xz\,\bar{j} + 2yz\,\bar{k}$ at the point $(1, -1, 1)$ then find

(a) $div\ f$

(b) $curl\ (curl\ f)$.

Or

16. (a) If F is a differentiable vector point function, then prove that $div(Curl\ F = 0)$.

(b) If $F = xy^2\bar{i} + 2x^2yz\,\bar{j} - 3yz^2\bar{k}$ then find $div\ F$ at (1, –1, 1).

17. Evaluate $\int_S F.N\,dS$ where $F = 18z\,\bar{i} - 12\bar{j} + 3y\,\bar{k}$ and S is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant.

Or

18. If $F = (2x^2 - 3z)\bar{i} - 2xy\,\bar{j} - 4x\bar{k}$ evaluate $\int_V \nabla . F\,dV$ where V is the closed region bounded by $x = 0, y = 0, z = 0, 2x + 2y + z = 4$.

19. Evaluate by Gauss divergence theorem for $\iint 4xz\,dy\,dz - y^2 dz\,dx + yz\,dx\,dy$ where S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

Or

20. State and prove Green's theorem in a plane.

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